SUPPORT VARIETIES OF $(\mathfrak{g},\mathfrak{k})$-MODULES OF FINITE TYPE

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1. Brief statement of results

Let $\mathfrak{g}$ be a semisimple Lie algebra over an algebraically closed field $\mathbb{F}$ of characteristic 0 and $\mathfrak{k}\subset\mathfrak{g}$ be a reductive in $\mathfrak{g}$ subalgebra.

Definition 1. A $(\mathfrak{g},\mathfrak{k})$-*module* is a $\mathfrak{g}$-module which after restriction to $\mathfrak{k}$ becomes a direct sum of finite-dimensional $\mathfrak{k}$-modules.

Definition 2. A $(\mathfrak{g},\mathfrak{k})$-module is of *finite type* if it is a $\mathfrak{k}$-module of *finite type*, i.e. has finite-dimensional $\mathfrak{k}$-isotypic components.

Let $\mathrm{Z}(\mathfrak{g})$ be the center of the universal enveloping algebra $\mathrm{U}(\mathfrak{g})$. Let $\chi:\mathrm{Z}(\mathfrak{g})\to\mathbb{F}$ be an algebra homomorphism.

Definition 3. We say that a $\mathfrak{g}$-module M *affords a central character* if for some homomorphism $\chi:\mathrm{Z}(\mathfrak{g})\to\mathbb{F}$ we have $zm=\chi(z)m$ for all $z\in\mathrm{Z}(\mathfrak{g})$ and $m\in M$.

Any simple $\mathfrak{g}$-module M affords a central character [5]. Let X be the variety of all Borel subalgebras of $\mathfrak{g}$. The category of $\mathfrak{g}$-modules which affords a central character χ is equivalent to the category of sheaves of $\mathcal{O}_X$-quasicoherent modules over the sheaf of twisted differential operators $\mathcal{D}^\lambda(X)$ for a suitable twist $\lambda\in\mathrm{H}^1(X,\Omega_X^{1,\,cl})$ [2], where $\Omega_X^{1,\,cl}$ is the sheaf of closed holomorphic 1-forms on X. In this category there is a distinguished full subcategory of holonomic sheaves of modules. Informally, holonomic sheaves of modules are $\mathcal{D}^\lambda(X)$-modules of minimal growth. The simple holonomic modules M are in one-to-one correspondence with the pairs (L,S), where L is an irreducible closed subvariety of X and S is a sheaf of $\mathcal{D}^\lambda(L')$-modules which is $\mathcal{O}(L')$-coherent after restriction to a suitable open subset $L'\subset L$. Moreover, a coherent holonomic module S is locally free on L' and one could think about it as a vector bundle S_B over L' with a flat connection. Note that flat local sections of this bundle are not necessarily algebraic.

Theorem 1. Let M be a finitely generated $(\mathfrak{g},\mathfrak{k})$-module of finite type which affords a central character. Then $\mathrm{Ind}M$ is a holonomic $\mathcal{D}^\lambda(X)$-module.

We also prove the following theorem(the necessary definitions see in the following section).

Theorem 2. Let $\mathcal{O}\subset\mathfrak{g}^*$ be a nilpotent coadjoint G-orbit, $\mathfrak{k}^\perp$ be the annihilator of $\mathfrak{k}$ in $\mathfrak{g}^*$, and $\mathrm{N}_\mathfrak{k}\mathfrak{g}^*$ be the $\mathfrak{k}$-null-cone in $\mathfrak{g}^*$. Then the irreducible components of $\mathcal{O}\cap\mathfrak{k}^\perp\cap\mathrm{N}_\mathfrak{k}\mathfrak{g}^*$ are isotropic subvarieties of $\mathcal{O}$.

Let $\mathrm{V}^{\cdot}_{\mathfrak{g},\mathfrak{k}}$ be the set of all irreducible components of possible intersections of $\mathrm{N}_K\mathfrak{k}^\perp$ with the G-orbits in $\mathrm{N}_G\mathfrak{g}^*$. This finite set of subvarieties of $\mathfrak{g}^*$ determines a finite set $\mathcal{V}^{\cdot}_{\mathfrak{g},\mathfrak{k}}$ of subvarieties of T^*X and a finite set $\mathrm{L}^{\cdot}_{\mathfrak{g},\mathfrak{k}}$ of subvarieties of X (see Definition 9 below).

Theorem 3. Let M be a finitely generated $(\mathfrak{g},\mathfrak{k})$-module of finite type which affords a central character and (L,S) be the corresponding pair consisting of a variety and a coherent sheaf as before. Then L is an element of $\mathrm{L}^{\cdot}_{\mathfrak{g},\mathfrak{k}}$.

2. Preliminaries

We work in the category of algebraic varieties over $\mathbb{F}$. By T^*X we denote the total space of the cotangent bundle of a smooth variety X and by T^*_xX the cotangent space to X at a point x. By $\mathrm{N}^*_{Y/X}\subset\mathrm{T}^*X|_Y$ we denote the conormal bundle to a smooth subvariety $Y\subset X$.

2.1. **D-modules versus $\mathfrak{g}$-modules.** Let $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}$ and $\Sigma \subset \mathfrak{h}$ be the root system of $\mathfrak{g}$ and Σ_+ be a set of positive roots. Denote by $\hat{\mathfrak{h}^*}$ the set of weights λ such that $\alpha^\vee(\lambda)$ is not a strictly positive integer for any positive root $\alpha^\vee$ of the dual root system $\Sigma^\vee \subset \mathfrak{h}^*$.

For a fixed λ we denote by $\mathcal{D}^\lambda(X)$ the sheaf of twisted differential operators on X and by $\mathrm{D}^\lambda(X)$ its space of global sections. The algebras $\mathrm{D}^\lambda(X)$ and $\mathrm{D}^\mu(X)$ are naturally identified if λ and μ lie in one shifted orbit of Weyl group [8]. Moreover, any such orbit intersects $\hat{\mathfrak{h}^*}$ [8]. If $\lambda \in \hat{\mathfrak{h}^*}$ the surjective homomorphism

$$\tau : \mathrm{U}(\mathfrak{g}) \to \mathrm{D}^\lambda(X)$$

identifies the category of quasicoherent $\mathrm{D}^\lambda(X)$-modules and the category of $\mathfrak{g}$-modules affording the central character $\chi = \chi_\lambda$. A. Beilinson and J. Bernstein have proved that both above categories are equivalent to the category of $\mathcal{D}^\lambda(X)$-modules:

$$\begin{array}{c|c} \text{Res: } \mathcal{D}^\lambda(X)\text{−mod} \longrightarrow \mathfrak{g}\text{-mod}^\chi & \text{Ind: } \mathcal{D}^\lambda(X)\text{-mod} \longleftarrow \mathfrak{g}\text{-mod}^\chi \\ \mathcal{F} \to \Gamma(X, \mathcal{F}) & M \otimes_{(1\otimes\tau)\mathrm{U}(g)} \mathcal{D}(X) \longleftarrow M \end{array} \quad .$$

In general Res and Ind identify the category $\mathfrak{g}$-mod$^\chi$ with a certain quotient of the category $\mathcal{D}^\lambda(X)$-mod. For more detailed exposition of the topic see for example [8].

2.2. **Three faces of the support variety.** The algebra $\mathrm{U}(\mathfrak{g})$ has a natural filtration such that gr $\mathrm{U}(\mathfrak{g})$=$\mathrm{S}(\mathfrak{g})$. The filtration on $\mathrm{U}(\mathfrak{g})$ induces a filtration on any finitely generated $\mathfrak{g}$-module M. We denote the associated graded $\mathrm{S}(\mathfrak{g})$-module by gr$M$. We denote the support of gr$M$ inside $\mathfrak{g}^*$ =Spec $\mathrm{S}(\mathfrak{g})$ by $\mathrm{V}(M)$.

A similar argument constructs for a $\mathcal{D}^\lambda(X)$-module $\mathcal{F}$ a positive cycle $\mathcal{V}(\mathcal{F})$, i.e. a formal linear combination of irreducible subvarieties of T^*X with positive integer coefficients.

Definition 4. The *singular support* $\mathcal{V}(M)$ of a simple $\mathfrak{g}$-module M is the cycle $\mathcal{V}(\mathrm{Ind}M)$ in T^*X.

Definition 5. The *support variety* $\mathrm{L}(M)$ of a simple $\mathfrak{g}$-module M is the projection of $\mathcal{V}(M)$ to X.

Let $\mathcal{X}$ be a G-variety for a reductive group G with Lie algebra $\mathfrak{g}$. The map ϕ :$\mathrm{T}^*\mathcal{X} \times \mathfrak{g} \to \mathbb{F}(\{(l,x),g\} \to l(gx), x \in X, l \in \mathrm{T}^*_x\mathcal{X}, g \in \mathfrak{g})$ determines a map $\phi_\mathcal{X}$:$\mathrm{T}^*\mathcal{X} \to \mathfrak{g}^*$ called the *moment map*. D.Barlet and M.Kashiwara [1], have proved that $\mathrm{V}(M) = \phi_X(\mathcal{V}(\mathrm{Ind}M))$. Therefore we have a diagram

$$\begin{tikzcd} & \mathcal{V}(M) \subset \mathrm{T}^*X \arrow[dl, "\phi_X"'] \arrow[dr, "\mathrm{pr}"] & \\ \mathrm{V}(M) \subset \mathfrak{g}^* & & \mathrm{L}(M) \subset X \end{tikzcd} \quad .$$

Lemma 1 ([10])**.** Let K be a reductive algebraic group with a Lie algebra $\mathfrak{k}$ and let X be an affine K-variety. Then $\mathbb{F}[X]$ is a $\mathfrak{k}$-module of finite type if and only if X contains finitely many closed K-orbits. In this case any irreducible component of X contains precisely one closed K-orbit.

Lemma 2. A finitely generated $(\mathfrak{g}, \mathfrak{k})$-module M is of finite type if and only if its associated variety $\mathrm{V}(M)$ has finitely many closed $\mathfrak{k}$-orbits. In this case the set of closed orbits consists just of the zero orbit.

Proof. Let J_M be the annihilator of $\mathrm{V}(M)$ in $\mathrm{S}(\mathfrak{g})$. Consider the $\mathrm{S}(\mathfrak{g})$-modules

$$\mathrm{J}_M^{-i}\{0\} := \{m \in \mathrm{gr}M \mid j_1...j_i m = 0 \text{ for all } j_1, ..., j_i \in \mathrm{J}_M \ \}.$$

One can easily see that these modules form an ascending filtration of gr$M$ such that $\cup_{i=1}^\infty \mathrm{J}_M^{-i}\{0\}$ =gr$M$. Since $\mathrm{S}(\mathfrak{g})$ is a Noetherian ring, the filtration stabilizes, i.e. $\mathrm{J}_M^{-i}\{0\}$ =gr$M$ for some i. By $\overline{\mathrm{gr}}M$ we denote the corresponding graded object. By definition, $\overline{\mathrm{gr}}M$ is an $\mathrm{S}(\mathfrak{g})/\mathrm{J}_M$-module. Suppose that $f\overline{\mathrm{gr}}M = 0$ for some $f \in \mathrm{S}(\mathfrak{g})$. Then $f^i\mathrm{gr}M = 0$ and hence $f \in \mathrm{J}_M$. This proves that the annihilator of $\overline{\mathrm{gr}}M$ in $\mathrm{S}(\mathfrak{g})/\mathrm{J}_M$ equals zero.

Suppose $\mathrm{V}(M)$ has a unique closed $\mathfrak{k}$-orbit. Let M_0 be a $\mathfrak{k}$-stable space of generators of $\overline{\mathrm{gr}}M$. Then there is a surjective homomorphism $M_0 \otimes_\mathbb{F} (\mathrm{S}(\mathfrak{g})/\mathrm{J}_M) \to \overline{\mathrm{gr}}M$. Since $\mathrm{V}(M)$ has finitely many closed $\mathfrak{k}$-orbits, $M_0 \otimes_\mathbb{F} (\mathrm{S}(\mathfrak{g})/\mathrm{J}_M)$ is a $\mathfrak{k}$-module of finite type. Therefore $\overline{gr}M$ is of finite type, which implies that M is a $(\mathfrak{g}, \mathfrak{k})$-module of finite type.

Assume now that M is of finite $\mathfrak{k}$-type. Set

$$\mathrm{Rad}\ M = \{m \in \overline{\mathrm{gr}}M \mid \text{ there exists } f \in \mathrm{S}(\mathfrak{g})/\mathrm{J}_M \text{ such that } fm = 0 \text{ and } f \neq 0\}.$$

Then Rad M is a proper $\mathfrak{k}$-stable submodule of $\overline{\mathrm{gr}}M$. Therefore there exists a finite-dimensional $\mathfrak{k}$-subspace $M_0 \subset \overline{\mathrm{gr}}M$ such that $M_0 \cap \mathrm{Rad}M=0$. The homomorphism $M_0 \otimes_{\mathbb{F}} (\mathrm{S}(\mathfrak{g})/\mathrm{J}_M) \to \overline{\mathrm{gr}}M$ induces an injective homomorphism $\mathrm{S}(\mathfrak{g})/\mathrm{J}_M \to M_0^* \otimes_{\mathbb{F}} \overline{\mathrm{gr}}M$. Therefore $\mathrm{S}(\mathfrak{g})/\mathrm{J}_M$ is of finite $\mathfrak{k}$-type and $\mathrm{V}(M)$ has only finitely many closed $\mathfrak{k}$-orbits. As $\mathrm{V}(M)$ is $\mathbb{F}^*$-stable, any irreducible component of it contains point 0 and this point is a closed $\mathfrak{k}$-orbit. □

Lemma 3 (S. Fernando [6])**.** If M is a finitely generated $(\mathfrak{g},\mathfrak{k})$-module then

$$\mathrm{V}(M) \subset \mathfrak{k}^{\perp} := \{x \in \mathfrak{g}^* : \forall k \in \mathfrak{k}\ x(k) = 0\}\ .$$

2.3. **Hilbert-Mumford criterion.** Let K be a reductive group, X be an affine K-variety, V be a K-module.

Theorem 4 (Hilbert-Mumford criterion)**.** The closure of any orbit $\overline{Kx} \subset X$ contains a unique closed orbit $K\overline{x} \subset X$. There exists a homomorphism $\mu : \mathbb{F}^* \to K$ such that $\lim\limits_{t\to 0} \mu(t)x = \bar{x} \in K\overline{x}$.

The *null-cone* $\mathrm{N}_{\mathfrak{k}}V := \{x \in V : 0 \subset \overline{Kx}\}$ is a closed algebraic subvariety of V [10].

Theorem 5. Let $x \in V$ be a point. Then $0 \in \overline{Kx}$ if and only if there exists a nonzero rational semisimple element $h \in \mathfrak{k}$ such that $x \in V_h^{>0}$, where $V_h^{>0}$ is the direct sum of h-eigenspaces in V with positive eigenvalues.

Corollary 1 ([10])**.** There exists a finite set H of rational semisimple elements of $\mathfrak{k}$ such that $\mathrm{N}_K V := \cup_{h\in H} KV_h^{>0}$, where $KV_h^{>0} := \{v \in V \mid v = kv_h$ for some $k \in K$ and $v_h \in V_h^{>0}\}$.

2.4. **Gabber's theorem.** Let G be the adjoint group of $\mathfrak{g}$ and M be a finitely generated $\mathfrak{g}$-module.

Definition 6. Suppose $\mathcal{X}$ is a smooth G-variety with a closed G-invariant nondegenerate 2-form ω. Such a pair $(\mathcal{X},\omega)$ is called a *symplectic G-variety*.

Definition 7. Let $(\mathcal{X},\omega)$ be a symplectic G-variety. We call a subvariety $Y \subset \mathcal{X}$
a)*isotropic* if $\omega|_{\mathrm{T}_y Y} = 0$ for a generic point $y \in Y$;
b)*coisotropic* if $\omega|_{(\mathrm{T}_y Y)^{\perp}} = 0$ for a generic point $y \in Y$;
c)*Lagrangian* if $\mathrm{T}_y Y = (\mathrm{T}_y Y)^{\perp}$ for a generic point $y \in Y$ or equivalently if it is both isotropic and coisotropic.

Theorem 6 (O. Gabber [7])**.** The variety $\mathrm{V}(M) \subset \mathrm{N}_G\mathfrak{g}^*$ is a coisotropic subvariety of $\mathfrak{g}^*$ with respect to the Kirillov symplectic structure. The variety $\mathcal{V}(M)$ is a coisotropic subvariety of T^*X with respect to the natural symplectic structure.

Definition 8. A finitely generated $(\mathfrak{g},\mathfrak{k})$-module M which affords a central character is called *holonomic* if $\tilde{V}$ is a Lagrangian subvariety of $G\tilde{V}$ for any irreducible component $\tilde{V}$ of $\mathrm{V}(M)$.

3. Proofs

Let $\mathfrak{g}$ be a semisimple Lie algebra and $\mathfrak{k} \subset \mathfrak{g}$ be a reductive in $\mathfrak{g}$ subalgebra. Let G be the adjoint group of $\mathfrak{g}$ and $K \subset G$ be a connected reductive subgroup such that lie $K = \mathfrak{k}$.

Theorem 2. Let $\mathcal{O} \subset \mathfrak{g}^*$ be a nilpotent coadjoint orbit, $\mathfrak{k}^{\perp}$ be the annihilator of $\mathfrak{k}$ in $\mathfrak{g}^*$, and $\mathrm{N}_{\mathfrak{k}}\mathfrak{g}^*$ be the $\mathfrak{k}$-null-cone in $\mathfrak{g}^*$. Then the irreducible components of $\mathcal{O} \cap \mathfrak{k}^{\perp} \cap \mathrm{N}_{\mathfrak{k}}\mathfrak{g}^*$ are isotropic subvarieties of $\mathcal{O}$.

Proof. As $\mathfrak{g}$ is semisimple, we can freely identify $\mathfrak{g}$ with $\mathfrak{g}^*$. Let h be a nonzero rational semisimple element of $\mathfrak{k}$. By definition $\mathfrak{g}_h^{\geq 0}$ is the direct sum of all h-eigenspaces in $\mathfrak{g}$ with nonnegative eigenvalues. Let $G_h^{\geq 0} \subset G$ be the parabolic subgroup with Lie algebra $\mathfrak{g}_h^{\geq 0}$, and let $S_G := G/G_h^{\geq 0}$ be a quotient. In the same way we define $\mathfrak{k}_h^{\geq 0}, K_h^{\geq 0}, S_K$. Let $\mathfrak{n}_h$ be the nilpotent radical of $\mathfrak{g}_h^{\geq 0}$. Obviously $Ke \subset S_G$ is isomorphic to S_K. By definition,

- $G\mathfrak{n}_h := \{x \in \mathfrak{g} \mid x = gn$ for some $n \in \mathfrak{n}_h, g \in G\}$,
- $K\mathfrak{n}_h := \{x \in \mathfrak{g} \mid x = kn$ for some $n \in \mathfrak{n}_h, k \in K\}$,
- $K(\mathfrak{n}_h \cap \mathfrak{k}^{\perp}) := \{x \in \mathfrak{g} \mid x = kn$ for some $n \in \mathfrak{n}_h \cap \mathfrak{k}^{\perp}, k \in K\}$.

Let $\phi : \mathrm{T}^*S_G \to \mathfrak{g}^*$ be the moment map. It is a straightforward observation that $G\mathfrak{n}_h$ coincides with $\phi(\mathrm{T}^*S_G)$, $K\mathfrak{n}_h$ coincides with $\phi(\mathrm{T}^*S_G|_{S_K})$, $K(\mathfrak{n}_h \cap \mathfrak{k}^{\perp})$ coincides with $\phi(\mathrm{N}^*_{S_K/S_G})$.

$$\begin{tikzcd}
\mathrm{T}^*S_G \arrow[d, "\phi"] & \mathrm{T}^*S_G|_{S_K} \arrow[l, "\text{inclusion}"'] \arrow[d, "\phi"] & \mathrm{N}^*_{S_K/S_G} \arrow[l, "\text{inclusion}"'] \arrow[d, "\phi"] \\
G\mathfrak{n}_h & K\mathfrak{n}_h \arrow[l, "\text{inclusion}"'] & K(\mathfrak{n}_h \cap \mathfrak{k}^{\perp}) \arrow[l, "\text{inclusion}"']
\end{tikzcd}$$

As $\mathrm{N}^*_{S_K/S_G}$ is an isotropic subvariety of T^*S_G, the variety $\phi(\mathrm{N}^*_{S_K/S_G})$ is isotropic in $G(\mathfrak{n}_h \cap \mathfrak{k}^\perp)$ and any subvariety $\tilde{V}$ of $K(\mathfrak{n}_h \cap \mathfrak{k}^\perp)$ is isotropic in $G\tilde{V}$. Therefore by Corollary 1 any subvariety $\tilde{V}$ of $\mathfrak{k}^\perp \cap \mathrm{N}_{\mathfrak{k}}\mathfrak{g}$ is isotropic in $G\tilde{V}$. □

Definition 9.
- Let $\mathrm{V}^\cdot_{\mathfrak{g},\mathfrak{k}}$ be the set of all irreducible components of all possible intersections of $\mathrm{N}_K\mathfrak{k}^\perp$ with a G-orbit in $\mathrm{N}_G\mathfrak{g}^*$.
- Let $\mathcal{V}^\cdot_{\mathfrak{g},\mathfrak{k}}$ be the set of all irreducible components of the preimages of $\mathrm{V}^\cdot_{\mathfrak{g},\mathfrak{k}}$ under the moment map $\mathrm{T}^*X \to \mathfrak{g}^*$.
- Let $\mathrm{L}^\cdot_{\mathfrak{g},\mathfrak{k}}$ be the set of all images of elements of $\mathcal{V}^\cdot_{\mathfrak{g},\mathfrak{k}}$ in X.

Theorem 3. Let M be a finitely generated $(\mathfrak{g},\mathfrak{k})$-module of finite type which affords a central character. The irreducible components of $\mathrm{V}(M)$ are elements of $\mathrm{V}^\cdot_{\mathfrak{g},\mathfrak{k}}$, the irreducible components of $\mathcal{V}(M)$ are elements of $\mathcal{V}^\cdot_{\mathfrak{g},\mathfrak{k}}$, the irreducible components of $\mathrm{L}(M)$ are elements of $\mathrm{L}^\cdot_{\mathfrak{g},\mathfrak{k}}$.

Proof. Let $\tilde{V}$ be an irreducible component of $\mathrm{V}(M)$ and $\mathcal{O}$ be the closure of $G\tilde{V}$. By Theorem 6 the variety $\tilde{V}$ is coisotropic. As $\tilde{V} \subset \mathrm{N}_K\mathfrak{k}^\perp \cap \mathcal{O}$, $\tilde{V}$ is isotropic, and therefore $\tilde{V}$ is Lagrangian and is an irreducible component of $\mathcal{O} \cap \mathrm{N}_K\mathfrak{g}^* \cap \mathfrak{k}^\perp$. □

Proof of Theorem 1. As any irreducible component $\tilde{\mathcal{V}}$ of $\mathcal{V}(M)$ is Lagrangian in T^*X, the module Ind$M$ is holonomic. □

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